Physics 131 Some extra problems for those that want them! You're welcome! Dr. AP

Newton's 1st law: What comes in a straight line at a constant speed, goes in a straight line at a constant speed (unless it interacts with something else) [Note: that constant speed may be the special case $v(t) = 0$.]

Newton's 2nd law: If the various interactions (forces) on something ***don't balance,*** then the vector sum of those interactions causes that something to accelerate an amount inversely proportional to its mass.

Newton's 3rd law: if something interacts with something else, the two objects experience that same force in opposite directions.

Newton's "0th" law: things only react to interactions *on* them, NOT *from* them, and ONLY while those interactions are occurring. ("object egotism")

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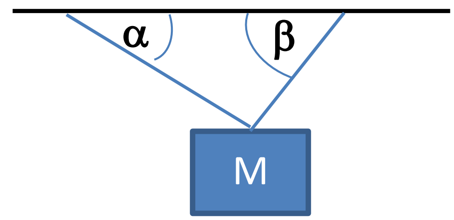
1. Classic "statics" problem. Imagine a box hanging from the ceiling in this unlikely way on two ropes:
   a. Draw the free body diagram for the box. Do not "break" any forces into components just yet.
   b. If α is 30° and β is 60°, what would be a clever choice of axes?
   c. If M is 10 kg, what are the tensions in the two ropes?



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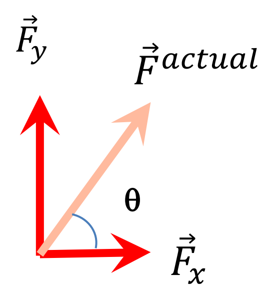
1. This is a corny setup where the third angle (call it γ?) is 90°. That makes the analysis a bit easier, as you'll see
   a. The FBD should simply show three forces: gravity pointing down, tension in the left rope pointing up and to the left at an angle α from the horizontal and tension in the right rope pointing up and to the right at an angle β from the horizontal; can you guess that the tension in the left rope is smaller?
   b. The two ropes are at 90° from each other, so you should probably line up your axes with the ropes so you only have to "break down" gravity into two components – one countering the left rope and one the right.
   c. Breaking down gravity, you can see that the right rope is "more" vertical, so gravity is mostly countering that…check your numbers later to make sure. In the meantime, a reminder of how to "break" down a vector into two (or more) components…

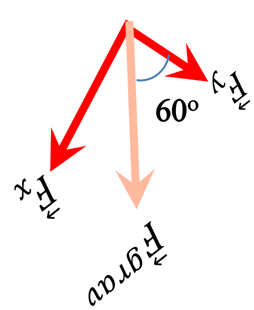
   The diagram at the right from lecture shows a pale arrow for the original vector and two dark red arrows for the components. The cosine of the angle between F_x and F_{actual} is the ratio of F_x divided by F_{actual}. So, rearranging:



$$\cos(\theta) = \frac{F_x}{F_{actual}} \ldots \qquad F_x = F_{actual}\cos(\theta)$$

   Careful: these are just the magnitudes, so if you need to account for direction, you can do that explicitly by thinking about the cosine and sine of the angle measured from the x-axis, or by inspection (I'd recommend you develop the habit of doing it by inspection, but maybe that's just me.)

   In our problem, $F_{gravity}$ is pointing down and we need to break it down into two pieces lined up with the left and right ropes, respectively. So it's like this diagram: By inspection of the geometry, which angle is which? Identifying "which angle goes where" is a skill that requires practice and your drawings should **emphasize** differences. (When the angles in question are 45°, this is less important since sin(45°) = cos(45°).) You can see by using your calculator that sin(30)=cos(60) and vice versa, so you can get the components of gravity as follows:

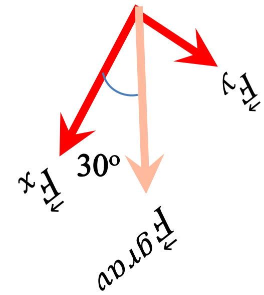


$$Mg\sin(60) = F_x^{grav}$$
$$Mg\cos(60) = F_y^{grav}$$

   Can you see that you get the same thing by complement? (see diagram next page)

$$Mg\cos(30) = F_x^{grav}$$
$$Mg\sin(30) = F_y^{grav}$$

Since the box isn't moving (a special case of no acceleration and coincidentally no velocity), then the net forces in each direction have to be zero:



$$\vec{F}_x^{net} = m\vec{a}_x \ldots \quad \vec{F}_x^{grav} + \vec{T}^{right\,rope} = 0 \ldots \quad Mg\sin(60) - T^{right\,rope} = 0$$

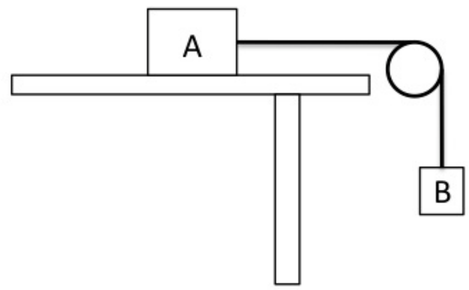
$$T^{right\,rope} = (10\text{ kg})(0.87)(10\text{ m/s}^2) = 87\text{ N}$$

And similarly (I'll let you fill in the steps for this one):

$$T^{left\,rope} = (10\text{ kg})(0.5)(10\text{ m/s}^2) = 50\text{ N}$$

One classic misleading thing about this problem's setup is that the rope on the left is longer, so people might mistake the tension in that rope to be larger than in the right rope. Instead you should think that since the two tensions "share" the fight against gravity, the more vertical rope is likely to take the brunt (the right rope).

2. Another classic problem. Block B hangs over a "frictionless" pulley on a rope which attaches to bock A. A weighs 6 lb, B weighs 4 lb.
   a. How fast does A accelerate from rest if the table is so slick there's almost no friction?
   b. What is the *minimum* coefficient of static friction between A and the table need to be so that the system is at rest?
   c. What is the coefficient of kinetic friction between A and the table need to be so that B and A move with constant speed?
   d. For the situation in c., can you figure out what that speed must be? If you can, solve for it; if you can't explain why not.



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3. a. The free body diagram for B is simple: weight pulling "down" and tension pulling "up"; how do we know the tension isn't sufficient to stop the system? Well, the free body diagram for A is also simple. In the vertical direction, there's gravity "down" and the normal force from the table "up" which *do* cancel. There is only a horizontal force on A of tension since we're assuming there's no friction, so it will accelerate to the "right" and B will accelerate "down" meaning that gravity wins *even if* B is lighter than A (so long as friction is negligible).
What we have here is a series of algebra to sort out. Let's choose "up" as positive *y*, and "right" as positive *x*. The vertical bit for A is dull, but for completion:
$$\vec{F}_A^{Net} = m_A\vec{a}_A \ldots in\ the\ y\ direction\text{:}\ \ \vec{F}_{Earth\ on\ A}^{gravity} + \vec{F}_{Table\ on\ A}^{Normal} = m_A\vec{a}_{Ay} = 0 \ldots \quad -m_A g\hat{\jmath} + N\hat{\jmath} = 0 \ldots \quad N = m_A g.$$
We've chosen to explicitly make *N* and *g* positive numbers by using + and – signs for i-hat and j-hat. As it happens, this last dull thing we've done won't be useful for part a. (but it will be important for b. and c. below).
The horizontal analysis of A (and corresponding connection to the vertical bit of B's motion) is what's interesting. The tension in the rope acts as a force "transmitter" and is to be assumed a massless rope unless otherwise specified. So it acts as our Newton "3rd law" connector between A and B! A feels it pull to the right and B feels it pull up because the pulley warps our sense of independent direction!
$$\vec{F}_A^{Net} = m_A\vec{a}_A \ldots in\ the\ x\ direction\text{:} \quad \vec{F}_{B\ on\ A}^{tension} = m_A\vec{a}_{Ax} \ldots \quad |T_{B\ on\ A}|\hat{\imath} = m_A a_{Ax}\hat{\imath}$$
Meanwhile, what's going on with B? Nothing in the horizontal direction. So we just have the *y* direction:
$$\vec{F}_B^{Net} = m_B\vec{a}_B \ldots in\ the\ y\ direction\text{:} \quad \vec{F}_{Earth\ on\ B}^{gravity} + \vec{F}_{A\ on\ B}^{tension} = m_B\vec{a}_{By} \ldots \quad |T_{A\ on\ B}|\hat{\jmath} - m_B g\hat{\jmath} = -m_B a_{By}\hat{\jmath} \ldots$$
Notice we have *explicitly chosen* the direction and sign of these forces so that the *"T"*s are just magnitudes. You should be cautious and consistent with such choices! So in the end, it boils down to these algebraic relations:
$$|T_{B\ on\ A}| = m_A a_{Ax}$$
$$|T_{A\ on\ B}| - m_B g = -m_B a_{By}$$
Here's where we argue that the rope's job is to transmit the tension (note, not *two* tensions), so in magnitude the *T*'s are the same! Furthermore, the accelerations of A and B are *together*, so we just have:
$$T = m_A a$$
$$T - m_B g = -m_B a$$
Which is just two equations, two unknowns (*T* and *a*). Solving it any old algebraic way you like, we get

$$a = \frac{m_B}{(m_A + m_B)} g \ldots \qquad a = \frac{4\text{ kg}}{10\text{ kg}} 10\text{ m/s}^2 = 4\text{ m/s}^2$$

Note that this analysis never plugged in numbers – and neither should you – until the VERY end. Note also that it doesn't matter if A weighs more than B or vice versa – test this at home with some string, a slick table and a couple of objects! You can forgo the pulley; just hang one mass over the edge of the table to develop your intuition. Notice also that our final formula can also be checked intuitively – if we increase the mass of B, the acceleration increases; if B is huge and A is tiny, the acceleration is almost the same as g itself! $\left(\frac{m_B}{(m_A+m_B)} \approx 1 \text{ if } m_A \ll m_B\right)$ and if B is tiny and A is huge, the acceleration gets small (and we'll also probably have to worry about friction). Again, test this at home with two very different mass objects and a piece of string.

b. We know from experience of things hanging off the edge of other things that if there is enough friction, the system *won't* slide. In essence, this problem is the same as part a. but with an acceleration of 0. The vertical analysis for A is identical, leading to $N = m_A g$. The horizontal analysis (I'm skipping steps here that you ought to fill out yourself) for A leads to:

$$T_{B\ on\ A} - f^{static}_{Table\ on\ A} = m_A a_{Ax} = 0 \ldots \quad T_{B\ on\ A} = f^{static}_{Table\ on\ A}$$
$$T_{A\ on\ B} - m_B g = -m_B a_{By} = 0 \ldots \quad T_{A\ on\ B} = m_B g$$

Now the static friction is at most proportional to the normal force by a constant related to the properties of the block A and the table surfaces. It's approximated by an empirical coefficient "mu" (static):
$f^{s}_{Table\ on\ A} \leq \mu_s N_{Table\ on\ A}$ (since there's only one normal force and one friction force, I'm going to throw caution to the winds and stop labeling them; you should probably not be so cavalier at first). We know from the vertical analysis on A that $N = m_A g$, so $f_s \leq \mu_s N = \mu_s m_A g$. From the horizontal analysis on A, we know that $T_{B\ on\ A} = f_s$, but by N3, we know that (numerically) $T_{B\ on\ A}$ and $T_{A\ on\ B}$ are third law pairs of equal magnitude; finally we know that $T_{A\ on\ B} = m_B g$ from the vertical analysis on B. Putting it all together and being **very, very sloppy with subscripts:**

$$m_B g = T = f_s \leq \mu_s N = \mu_s m_A g$$
$$m_B g \leq \mu_s m_A g \ldots \quad \mu_{static} \geq m_B / m_A$$

As a check on your work, do you think the friction coefficient (and therefore the friction) should be larger if the mass of B increases? (yes) Furthermore, can it be smaller if the mass of A increases? (yes). Does "mu" have units? (no)

c. and d. Let's answer d. first: analysis of forces only tells us about acceleration, so the answer is no. The acceleration is zero, so the constant speed (whatever it is) doesn't change. As for c., analyzing this without realizing that a constant speed of 0 is simply a special case of a constant speed, means wasting valuable time. You'll have exactly the same kind of analysis as in b. except that we're talking about $\mu_{kinetic}$ which does not involve *inequalities*. So (leaving this as an exercise for the student) you should get $\mu_{kinetic} = m_B / m_A$.